




Original Article

Analysis of Static & Dynamic Performance Parameters of TOR-VCube Hybrid Parallel Interconnection Connection Network for High Performance Computing

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ABSTRACT

In this paper we analyse the topological properties of a new hybrid scalable parallel interconnection network termed as Tor-Vcube which serves as the backbone network for today's high performance computing in all most all application areas like chemical, pharmacy, health, human interaction industries. Through this network the various computer components communicate with one another, its topology is one of the most crucial factors to take into account when designing high-performance computers. The flawless data transfer between the source and destination nodes is determined by the topology's characteristics, including connection, dependability, cost, fault tolerance, diameter, and bisection width. In this research, we present TOR-VCube (TVC), a new hybrid interconnection network topology that is a product of two well-known traditional interconnection topologies: Torus and Varietal hypercube. Additionally, we demonstrate the design and features of the suggested connectivity topology. Our findings indicate that this topology is not suitable but perfect computational network for health industry. Additionally, we demonstrate the design and features of the suggested interconnection topology. We have also outlined several key properties and developed two routing algorithms for TVC. Our findings indicate that the proposed interconnection topology exhibits high connectivity, reduced diameter, lower cost, fault tolerance, scalability, and minimized average distance. The various reliability metrics of TVC have been calculated and shown to surpass those of its counterpart topologies and its parent topologies.

Keywords: High Performance Computing, Node degree, Message Traffic Density, Cost Factor, Routing Function.

INTRODUCTION

The interconnection network plays a vital role in the design of high performance computing systems as it defines the means of data exchange among many standalone processing units. The interconnection network also presents the underlying architecture i.e. topology used for these systems. Hence, the performance of these systems mainly depends on their interconnection network. Thus, in order to design a highly reliable high performance computing system, it must be ensured that the used interconnection network should be highly reliable i.e. it should work satisfactorily for a specific period of time even occurrence of some failures like link and/or computing units. Further the use of interconnection network in these systems should be economical. One way to measure the cost of the interconnection network is the product of its degree to diameter. In other words, it can be said that highly reliable as well as economical interconnection network must be deployed to design such systems. Some examples of Interconnection network include Hypercube and its variants, Torus, Mesh, eTVC. The hypercube (HC) has been used in [1-2, 20, 28-31] due to its properties like small diameter, strong connectivity regularity, symmetry, recursive construction, partitionability, fault tolerance, and reliability. Many variants of hypercube have been proposed in the past either to enhance its reliability or to decrease its cost. The variant includes cube connected cycles [23], twisted hypercube [3], folded hypercube [5], crossed cube [6, 8], exchanged hypercube [4,41], incomplete crossed cube [5], banyan hypercube [15], varietal hypercube eTVC. The hypercube is also a good choice of researchers to propose many hierarchical interconnection networks viz. hierarchical hypercubes, hierarchical crossed cubes, hierarchical

cube networks, extended hypercube, eTVC. [30]. However, the main problem associated with hypercube and its variants is their implied constraint to scale. The prominent topologies to provide a high degree of scalability are Mesh and torus [24-27]. These networks are found to be used widely in the area of scientific calculations, flow dynamics, structural analysis, eTVC. However, the large diameter of mesh network prohibits its use in the design of a massively parallel computer system [9-10]. In addition to this, high cost and difficulty in set-up and maintenance of this network further make it difficult to implement in real life parallel machines. In comparison to mesh, Torus has high connectivity and more reliable. Hence, it is used in the design of various parallel machines such as Alpha 21364-based HPGS 1280 [4], Cray XT3 [11] and XT4 [12], X1E [13], Cray X1-E vector computer [14]. Moreover, IBM Blue Gene family of massively parallel computers are some remarkable examples of mixed Radix 3D Tori [30-32]. In order to decrease very large diameter of the torus, a new class of recursively structured torus connections networks called recursive diagonal torus (RDT) has been proposed in [22]. The discussion carried out so far reveals that the torus has a fixed degree but has a relatively large diameter ($2n$). While, the diameter of the hypercube is comparatively small, but its node connectivity increases logarithmically with the size of the network. Under such conditions taking the product of two standard topologies is a potential process of constructing a new interconnection network [16-19]. The cross product of interconnection networks is observed to be outperforming traditional topologies with different structural properties [11]. Hence, the problem of designing a scalable, cost effective, highly reliable, fault tolerant network with the lower diameter and strong connectivity with less hardware complexity is yet a challenge. This motivates us to propose a new hybrid interconnection network called TOR- VARIETAL CUBE (TVC) which inherits the properties of two popular interconnection topologies: varietal hypercube and torus. The proposed topology (TVC) exploits most of the important properties of hypercube and torus. The present work focuses and analyzes the different design factors of TOR-VCUBE (TVC) and compares its topological properties with that of other important interconnection topologies.

PROPOSED INTERCONNECTION TOPOLOGY: TOR-VCube (TVC)

The proposed interconnection network uses an n-dimensional hypercube as its base. Additional links from a torus interconnection network are added to the base Varietal hypercube interconnection network to form an n-dimensional TOR-Varietal CUBE. e.g. For a 3D TVC, first a 3D Varietal hypercube (VHC) is taken (Fig. 1), and then extra 4 links from the torus interconnection topology are added to it. The extra links are connected to the adjacent opposite nodes in torus pattern (Fig. 2). The proposed topology is scalable and therefore is most suitable for large scale parallel processing (Fig. 3).

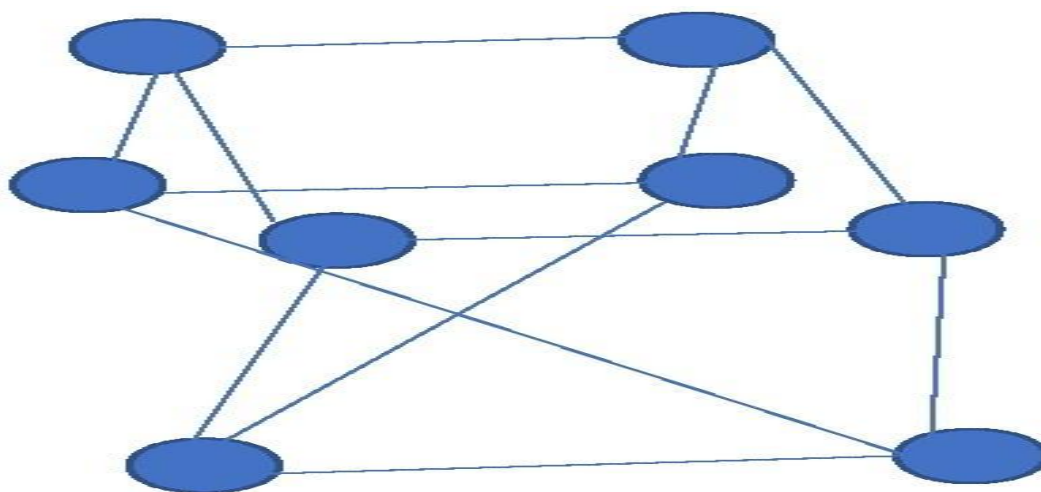


Fig. 1 Varietal Hypercube

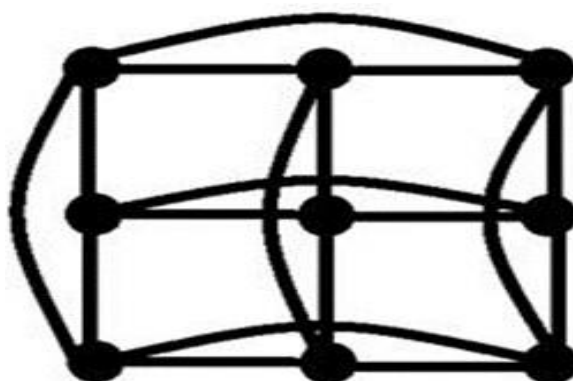


Fig. 2 Torus

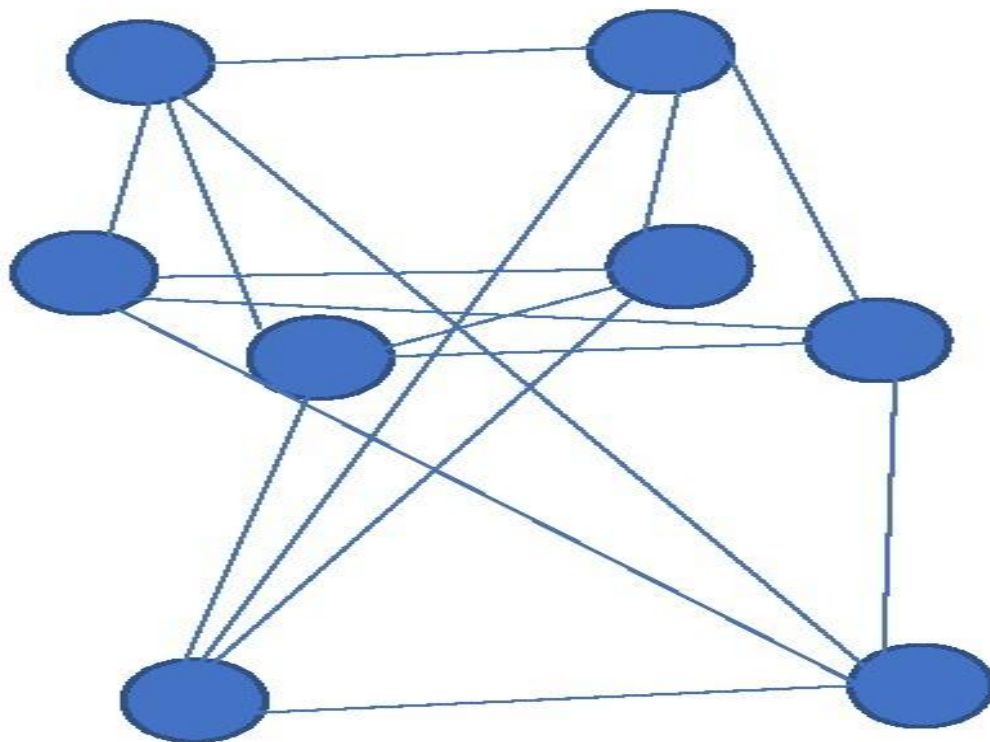


Fig. 3 TOR Vcube (TVC) of Dimension 3

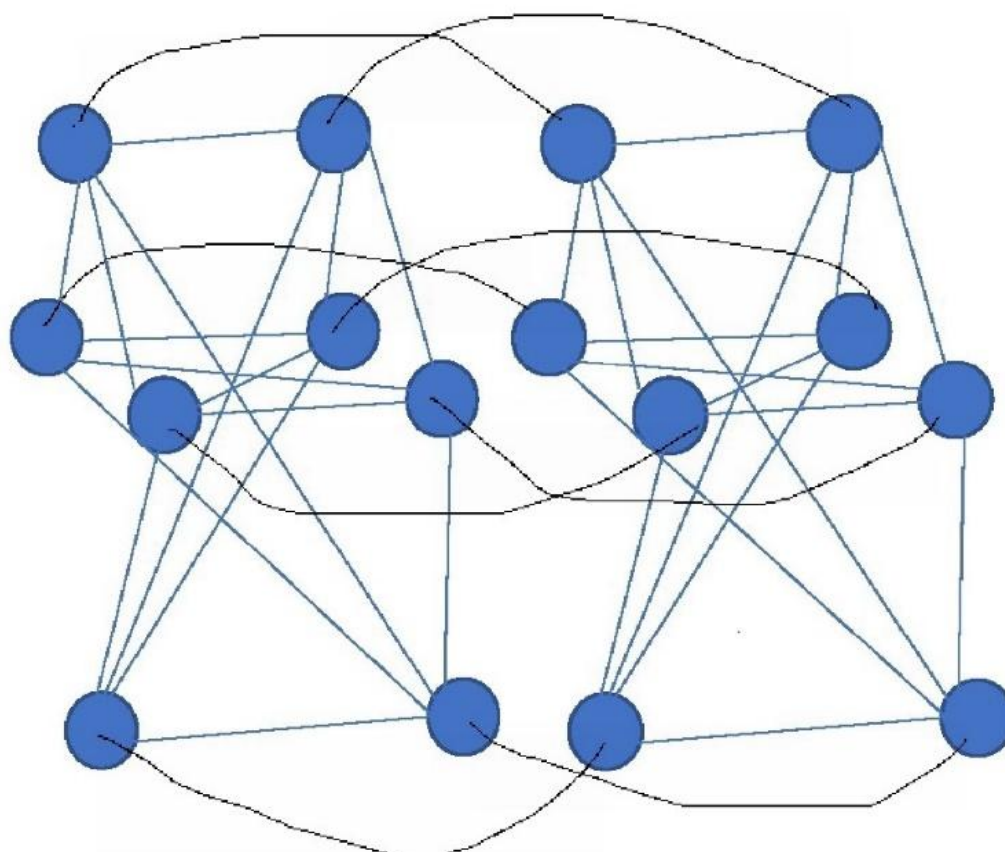


Fig. 4 TOR Vcube (TVC) of Dimension 4

TOPOLOGICAL PROPERTIES OF TOR-VCUBE INTERCONNECTION NETWORK

The TOR-VCUBE is viewed as an undirected graph $G(V, E)$, where V is the node set representing the processing elements and E represents the set of edges representing the connecting links between the processing elements. The different topological properties of TOR-VCube are presented in this Section.

Nodes

The number of nodes of n-dimensional TOR-VCUBE is 2^n .

Degree

The degree of an interconnection network is the degree of the node which has a maximum number of links connected to it. The degree of TOR-VCUBE is $(n+1)$ since every node is connected to exactly $n+1$ of its neighbor nodes.

Links

A link denotes a communication edge between two processing elements in a network. The nodes **u** and **v** are said to be adjacent if a link $l = (u, v) \in E$.

Theorem 1: The total number of links of n-dimensional TOR-VCUBE is $(n+1)2^{n-1}$

Proof: Let TOR-VCUBE is an undirected graph $G(V, E)$ consisting of 2^n number of nodes,

If L is the total number of links of TOR-VCUBE (V, E) having dimension 'n' then the L can be computed by using Handshaking Lemma of Graph Theory.

Further the degree of TOR-VCUBE is $(n+1)$ since every node is connected to exactly $n+1$ of its neighbor nodes.

By Handshaking Lemma of Graph Theory

Sum of the degree of vertices = 2 * Number of edges of the graph

$$2L = (n+1) * 2^n$$

$$L = \{(n+1) * 2^n\} / 2$$

$$= (n+1) * 2^{n-1}$$

Diameter

The diameter of graph G ($D(G)$), is the maximum of the minimum distances between any two distinct vertices $d(u, v)$ of G. This is measured in terms of a number of distinct hops between any two nodes. It determines the number of hops a node will take to broadcast a message to all other nodes in the network. Therefore, it should be kept small.

Theorem 2 : The diameter of the TOR-VCUBE is $\lceil 2n/3 \rceil - 1$

Proof: Since every node is connected to its neighboring nodes by $n+1$ links, So the distance to nonadjacent node of TOR-VCUBE is $\lceil 2n/3 \rceil - 1$ as the diameter of varietal hypercube is $\lceil 2n/3 \rceil$

Cost

For a symmetric interconnection network, the cost is defined as the product of the degree and the diameter of the network. This is an important factor is broadly used in performance evaluation.

Mathematically,

$$\text{Cost} = \text{degree} \times \text{diameter}$$

Therefore, the cost of TOR-VCUBE can be computed as follows :

$$\text{Cost} = \text{Degree} * \text{Diameter}$$

$$= (n+1) * \lceil 2n/3 \rceil - 1$$

As the degree of TOR-VCUBE is $(n+1)$ and the diameter of the TOR-VCUBE is $\lceil 2n/3 \rceil - 1$

Lemma 1 : $D_{\text{TOR-VCube}} < D_{\text{VHC}} < D_{\text{TOR-Cube}} < D_{\text{HC}}$

where $D_{\text{TOR-VCube}}$ is the diameter of TOR-VCube TVC(n,k)

D_{VHC} is the diameter of Varietal Hyper Cube VHC(n,k)

D_{HC} is the diameter of Hyper Cube HC(n,k)

Proof: The diameter of TOR-VCUBE is $\lceil 2n/3 \rceil - 1$, the diameter of Varietal Hyper Cube VHC(n,k) is $\lceil 2n/3 \rceil$ and the diameter of Hyper Cube HC(n,k) is n .

Lemma 2 : $\text{Cost}_{\text{TOR-VCube}} < \text{Cost}_{\text{TOR-Cube}}$

where $\text{Cost}_{\text{TOR-VCube}}$ is the cost factor of TOR-VCube TVC(n,k)

$\text{Cost}_{\text{TOR-Cube}}$ is cost factor of TOR-Cube TC(n,k)

Proof: The cost of cube network is given by the product of the degree of the node and the diameter. As the the diameter of TOR-VCube is $\lceil 2n/3 \rceil - 1$ and the degree of the node is $n+1$. Hence The cost of TOR-VCube is $\{(\lceil 2n/3 \rceil - 1 \} * (n+1)$. As the diameter of TOR-Cube is $n-1$ and the degree of the node is $n+1$. Hence the cost of TOR-Cube is $\{(n-1) * (n+1)\}$.

At level 1 we have $2^{n(k-1)}$ NC at one side and also same number of NCs at opposite side as seen from the construction

According to the Induction Hypothesis P(m) is true .

$$P(m) : 2 * 2^{n(k-m)}$$

At level m we have $2 * 2^{n(k-m)}$ NC

Induction Step : As we go higher the number of PE/NC is divided 2^n

$$\begin{aligned} \text{The number of NC at level } m+1 &= \text{Number of NC at level } m / 2^n \\ &= 2 * 2^{n(k-m)} / 2^n \\ &= 2 * 2^{n(k-m)-n} \\ &= 2 * 2^{n(k-m-1)} \end{aligned}$$

Message Traffic Density

An effective interconnection network must have extensive bandwidth to incorporate the consequential traffic in order to keep the traffic density minimum. If every node is sending message to another node at distance 'd' apart, then it is required to analyze the performance of the network in managing the message traffic. The message traffic density is

$$\text{defined by } \frac{\text{Averagedistance*Numberofnodes}}{\text{Totalnumberlinks}}$$

Theorem 5: The message traffic density of TOR-VCUBE (TVC) is $\frac{dN_t}{L}$

Where $N_t = N + M$ and L is the number of links

Proof : Assuming that each node is sending one message to a node at distance d on the average and considering the availability of n links to accommodate such a traffic, $\frac{dN_t}{L}$ can be a good measure to estimate message traffic in the network

As we know that $N = 2^{nk}$ and $M = (2^{nk} - 1) / (2^n - 1)$

At level 1 we have $2 * 2^{n(k-1)}$ NCs and at level k there are only two NCs . So the sum of processors from level 1 to k-1 is

$$2 * \{ [(2^{nk} - 1) / (2^n - 1)] - 2^{n(k-1)} - 1 \}$$

Now assuming handshaking lemma from graph i.e.

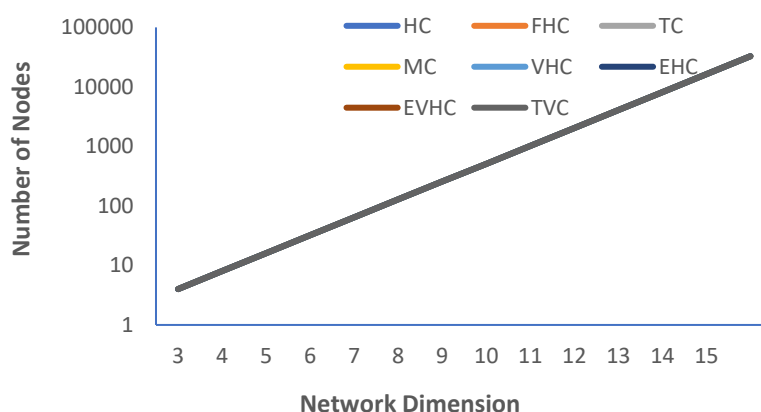
$2(\text{Number of edges of the graph}) = \text{Sum of the degrees of vertices}$

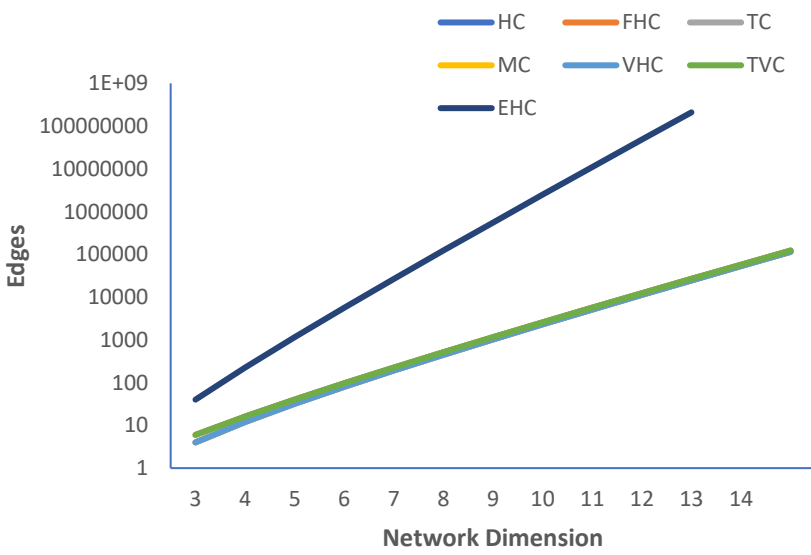
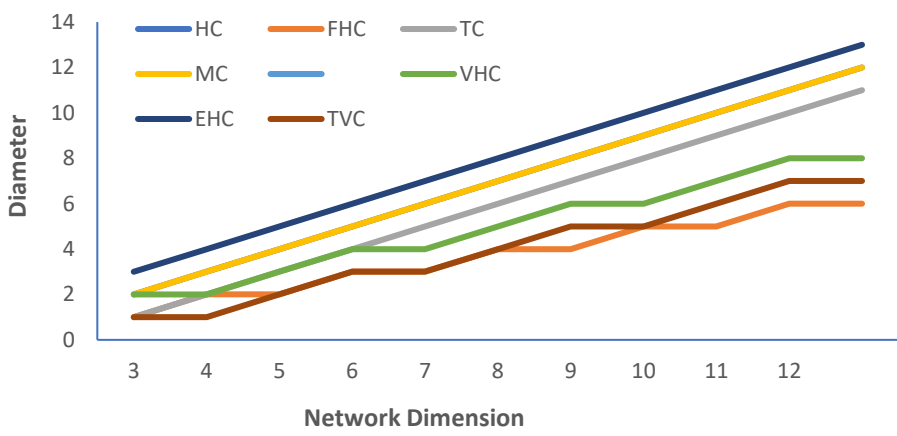
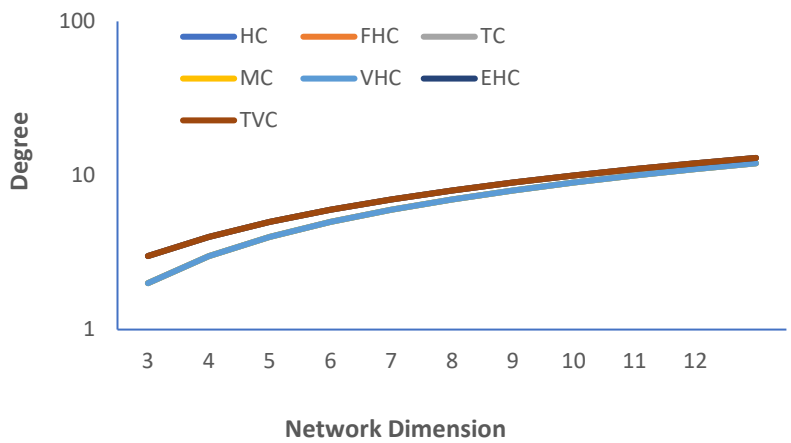
We have ,

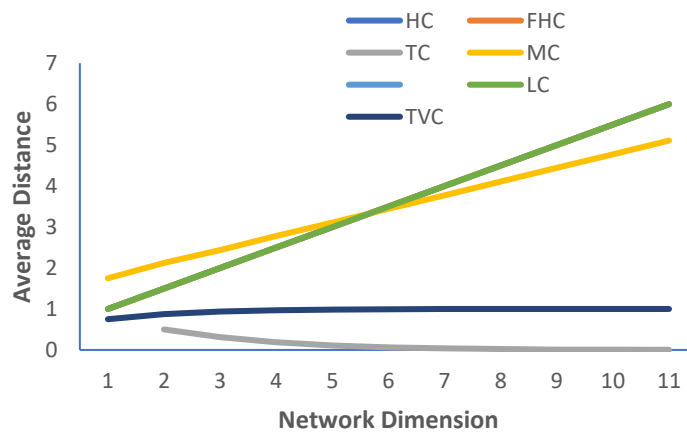
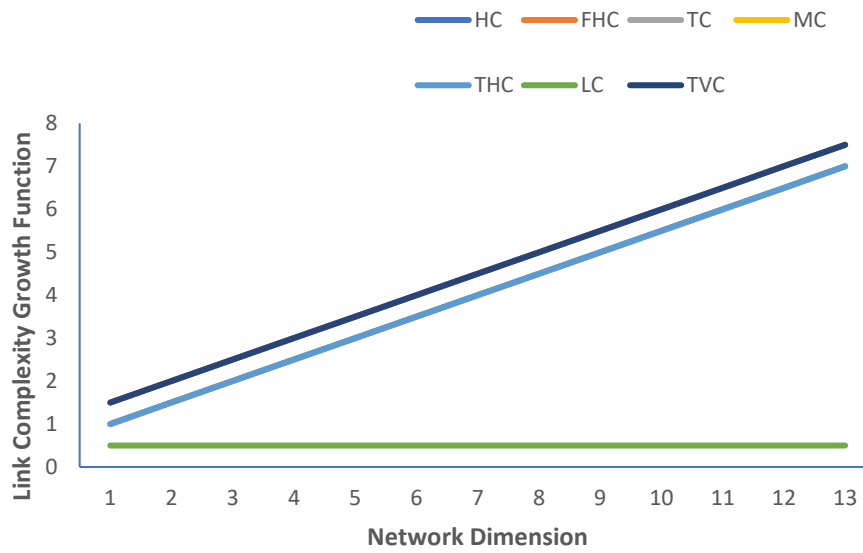
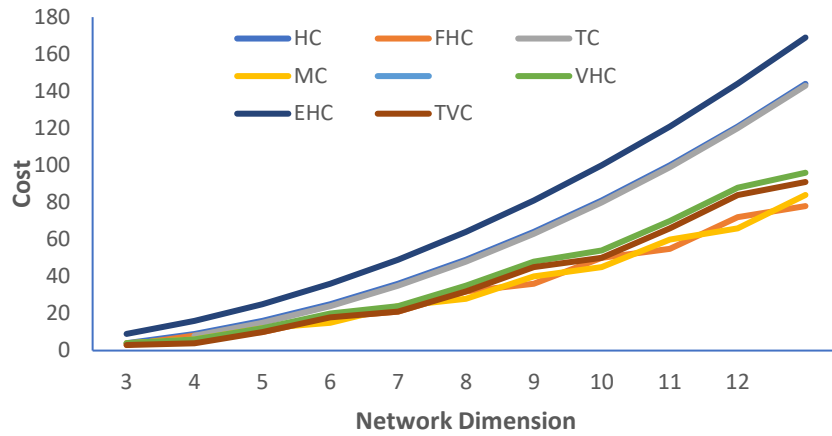
$$\begin{aligned} 2L &= 2^{nk} (n + 2) + 2 * \{ [(2^{nk} - 1) / (2^n - 1)] - 2^{n(k-1)} - 1 \} (2^n + n + 3) + 2 * 2^{n(k-1)} * (2^n + n + 2) + 2 * 2^{n+1} \\ L &= 2^{nk-1} (n + 2) + \{ [(2^{nk} - 1) / (2^n - 1)] - 2^{n(k-1)} - 1 \} (2^n + n + 3) + 2^{n(k-1)} * (2^n + n + 2) + 2^{n+1} \end{aligned}$$

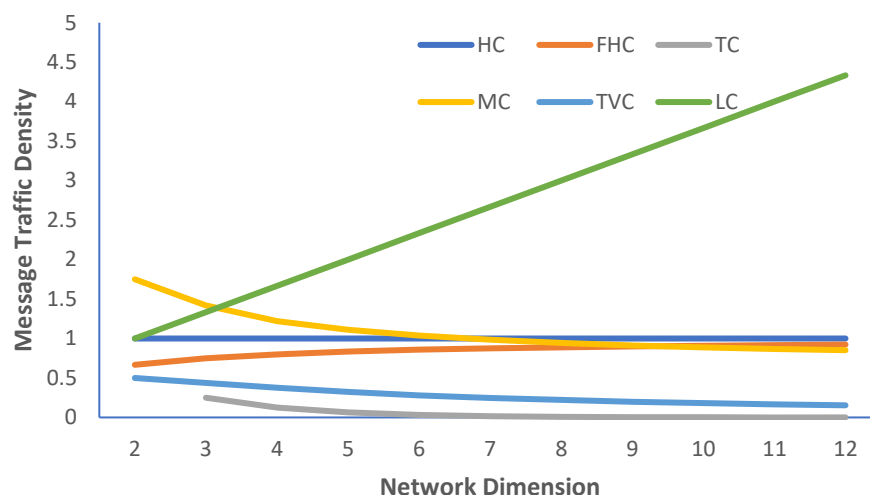
RESULTS

The following Table for message density of ECC and Table for message density of TVC shows the simulated result for the values of d , N , M , L and $\frac{dN_t}{L}$ for ECC(3,k) and TVC(3,k) basing on the given proposition.









The above table depicts that TVC has less message traffic density not only from its parent topologies but also from some other important topologies. Therefore, the hardware complexity for the design of TVC is also economical, reasonable and feasible.

CONCLUSIONS AND FUTURE SCOPE

A new interconnection topology namely TOR-VCUBE (TVC) is proposed in this paper. The proposed topology is inherited from two base class topologies: Varietal hypercube and torus. It can be observed that the proposed interconnection network is more suitable for parallel computer architecture because of its high connectivity, lesser diameter, low cost, more fault tolerant and low average distance. It can be concluded that the proposed interconnection network is highly reliable and cost effective with a better degree of scalability. It can be concluded that the proposed interconnection network is highly reliable and cost effective with a better degree of scalability. The different reliability measure of the TVC like network reliability, two-terminal reliability and k-terminal reliability can be evaluated and compared against that of other interconnection networks of interest in future. We can also study the broadcasting algorithms in near future. The work carried out in this paper may further be extended to propose a new hierarchical interconnection network using TOR-VCUBE as the base network.

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